QUESTION SET

01. the relation between price (P) and demand (D) of a cup of tea is given as
$$D = \frac{32}{P}$$
Find the rate at which the demand changes when the price is Rs 4/- . Interpret the result
ans : −2

02. the demand D for a price P is given as
$$D = \frac{27}{P}$$
Find the rate of change of demand when price is 3
ans : −3

03. the demand D of biscuits at price P is given as $D = \dfrac{64}{P^3}$
Find the marginal demand , when price is Rs 2/−
ans : −12

04. for a commodity , price − demand relationship is given as
$$D = \frac{P + 5}{P - 1}$$
Find marginal demand when price is 2
ans : −6

05. the demand function of a commodity is given as $P = 20 + D - D^2$. Find the rate at which price is changing when demand is 3
ans : −5

06. the relation between supply S and price P of a good is given as $S = 2P^2$. Find the marginal supply at price Rs 5/−. Interpret the result
ans : 20

07. The supply S of electric bulbs at price P is given by $S = 2P^3 + 7$
Find the marginal supply when price is 5/− . Interpret the result
ans : 150

08. the supply S for a commodity at price P is given by $S = P^2 + 9P - 2$
Find the marginal supply when price is 7
ans : 23

09. For a commodity ,
demand $D = \dfrac{24P}{P - 2}$ and
Suppy $S = P^2$
Find equilibrium price . Find marginal demand and marginal supply at that price
ans : 6 , −3 , 12

10. the demand function of a commodity is given by $P = 32 + 3D - D^2$
Find the rate at which the price is changing , when demand is 2 . Also find the rate at which the total Revenue R is changing at that time
ans : −1 , 32

11. the demand function is given as
$$P = 175 + 9D + 25D^2$$
Find the total revenue and marginal revenue when demand is 10
ans : 27650 , 7855

12. The price P and the demand x of pens has relation $x = 20\sqrt{P} - 4$
Find the marginal revenue when P = 9
ans : 86

13. the cost C for an output x is given as
$$C = x^4 - 2x^3 + 80x + 150$$
Find the rate at which the cost is changing when output x is 2
ans : 88

14. The total cost of producing x items is given by $C = x^2 + 4x + 4$
Find the average cost and the marginal cost .
What is the marginal cost when x = 7
ans : x + 4 + 4/x , 2x + 4 , 18

15. the cost of producing x – articles is given by
$$C = x^2 + 15x + 81$$
Find the average cost and marginal cost functions . Find the marginal cost when x = 10 . Find x for which the marginal cost equals average cost
ans : x+15+81/x , 2x+15 , 35 , 9

16. if the total cost function is given by ;
$$C = 5x^3 + 2x^2 + 7$$
Find the average cost and the marginal cost when x = 4
ans : 359/4 , 256

17. the total cost of x pencils is given by
$$C = 15 + 28x – x^2$$
Find x when the marginal cost is 20 . Find the average cost at this value of x
ans : 4 , 111/4

18. the total cost of producing n note books is given by
$$C = 1500 – 75n + 2n^2 + \frac{n^3}{5}$$
Find the marginal cost at n = 10
ans : 25

19. the total cost of 't' toy cars is given by
$$C = 5(2^t) + 17$$
Find the marginal cost and average cost at t = 3
ans : 40.log2 , 19

SOLUTION SET

01. the relation between price (P) and demand (D) of a cup of tea is given as

$$D = \frac{32}{P}$$

Find the rate at which the demand changes when the price is Rs 4/- . Interpret the result

SOLUTION

$$D = \frac{32}{P}$$

Rate of change of demand when P = 4

$$= \frac{dD}{dP}$$

$$= \frac{-32}{P^2}$$

Put p = 4

$$= \frac{-32}{16}$$

$$= -2 \quad , \quad \text{Demand falls at p = 4}$$

02. the demand D for a price P is given as

$$D = \frac{27}{P}$$

Find the rate of change of demand when price is 3

SOLUTION

$$D = \frac{27}{P}$$

Rate of change of demand when P = 4

$$= \frac{dD}{dP}$$

$$= \frac{-27}{P^2}$$

Put p = 3

$$= \frac{-27}{9}$$

$$= -3$$

03. the demand D of biscuits at price P is given as $D = \frac{64}{P^3}$

Find the marginal demand , when price is Rs 2/–

SOLUTION

$$D = \frac{64}{P^3} = 64P^{-3}$$

Marginal demand at P = 2

$$= \frac{dD}{dP}$$

$$= 64(-3P^{-4})$$

$$= \frac{-192}{P^4}$$

Put P = 2

$$= \frac{-192}{16}$$

$$= -12$$

04. for a commodity , price – demand relationship is given as

$$D = \frac{P + 5}{P - 1}$$

Find marginal demand when price is 2

SOLUTION

$$D = \frac{P + 5}{P - 1}$$

Marginal Demand when p = 2

$$= \frac{dD}{dP}$$

$$= \frac{(P-1)\frac{d}{dP}(P+5) - (P+5)\frac{d}{dP}(P-1)}{(P-1)^2}$$

$$= \frac{(P-1).1 - (P+5).1}{(P-1)^2}$$

$$= \frac{p - 1 - p - 5}{(P-1)^2}$$

$$= \frac{-6}{(P-1)^2.}$$

Put p = 2

$$= -6$$

05. the demand function of a commodity is given as $P = 20 + D - D^2$. Find the rate at which price is changing when demand is 3

SOLUTION :

$P = 20 + D - D^2$

Rate of change of price when D = 3

$= \dfrac{dP}{dD}$

$= 1 - 2D$

Put D = 3

$= 1 - 6$

$= -5$

06. the relation between supply S and price P of a good is given as $S = 2P^2$. Find the marginal supply at price Rs 5/–. Interpret the result

SOLUTION :

$S = 2P^2$

Marginal supply at P = 5

$= \dfrac{dS}{dP}$

$= 4P$

Put P = 5

$= 20$; supply increases with price

07. The supply S of electric bulbs at price P is given by $S = 2P^3 + 7$
Find the marginal supply when price is 5/– . Interpret the result

SOLUTION :

$S = 2P^3 + 7$

Marginal supply at P = 5

$= \dfrac{dS}{dP}$

$= 6P^2$

Put P = 5

$= 6(25)$

$= 150$; supply increases with price

08. the supply S for a commodity at price P is given by $S = P^2 + 9P - 2$
Find the marginal supply when price is 7

SOLUTION :

$S = P^2 + 9P - 2$

Marginal supply at P = 5

$= \dfrac{dS}{dP}$

$= 2P + 9$

Put P = 7

$= 14 + 9$

$= 23$

09. For a commodity ,
demand $D = \dfrac{24P}{P - 2}$ and

Suppy $S = P^2$
Find equilibrium price . Find marginal demand and marginal supply at that price

SOLUTION :

EQUILIBRIUM PRICE

$D = S$

$\dfrac{24P}{P - 2} = P^2$

$\dfrac{24}{P-2} = P$

$24 = P(P - 2)$

$6(4) = P(P - 2)$ $\therefore P = 6$

$D = \dfrac{24P}{P-2}$

Marginal Demand at P = 6

$= \dfrac{dD}{dP}$

$= \dfrac{(P-2)\dfrac{d}{dP}24P - 24P\dfrac{d}{dP}(P-2)}{(P-2)^2}$

$= \dfrac{(P-2).24 - 24P(1)}{(P-2)^2}$

$= \dfrac{24P - 48 - 24P}{(P-2)^2}$

$= \dfrac{-48}{(P-2)^2}$

Put P = 6

$= \dfrac{-48}{(6-2)^2}$

$= \dfrac{-48}{16}$ $= -3$

$S = P^2$

Marginal Supply at P = 6

$= \dfrac{dS}{dP}$

$= 2P$

   put P = 6

$= 12$

10.   the demand function of a commodity is given by $P = 32 + 3D - D^2$
Find the rate at which the price is changing , when demand is 2 . Also find the rate at which the total Revenue R is changing at that time

SOLUTION

$P = 32 + 3D - D^2$

Rate of change of price when D = 2

$= \dfrac{dP}{dD}$

$= 3 - 2D$

   Put D = 2

$= 3 - 4$

$= -1$

TOTAL REVENUE

$R = pD$

$= (32 + 3D - D^2).D$

$= 32D + 3D^2 - D^3$

Rate of change of total Revenue R when D = 2

$= \dfrac{dR}{dD}$

$= 32 + 6D - 3D^2$

   Put D = 2

$= 32 + 12 - 3(4)$

$= 32 + 12 - 12$

$= 32$

11.   the demand function is given as
     $P = 175 + 9D + 25D^2$
Find the total revenue and marginal revenue when demand is 10

SOLUTION

Total Revenue

$R = pD$

$= (175 + 9D + 25D^2).D$

$= 175D + 9D^2 + 25D^3$

   Put D = 10

$= 1750 + 900 + 25000$

$= 27650$

Marginal Revenue when D = 10

$= \dfrac{dR}{dD}$

$= 175 + 18D + 75D^2$

   Put D = 10

$= 175 + 180 + 7500$

$= 7855$

12.   The price P and the demand x of pens has relation $x = 20\sqrt{P} - 4$
Find the marginal revenue when P = 9

SOLUTION :

Total Revenue

$R = p.x$

$= P(20\sqrt{P} - 4)$

$= 20P\sqrt{P} - 4P$

$= 20P^{3/2} - 4P$

Marginal Revenue when P = 9

$= \dfrac{dR}{dP}$

$= 20 . \dfrac{3}{2} P^{1/2} - 4$

$= 30\sqrt{P} - 4$

   Put P = 9

$= 30(3) - 4$

$= 86$

13.   the cost C for an output x is given as
     $C = x^4 - 2x^3 + 80x + 150$
Find the rate at which the cost is changing when output x is 2

SOLUTION

Rate of change of cost when x = 2

$= \dfrac{dC}{dx}$

$$= 4x^3 - 6x^2 + 80$$

Put x = 2

$$= 4(8) - 6(4) + 80$$

$$= 32 - 24 + 80$$

$$= 88$$

14. The total cost of producing x items is given by $C = x^2 + 4x + 4$
Find the average cost and the marginal cost .
What is the marginal cost when x = 7

SOLUTION :

$C = x^2 + 4x + 4$

Average Cost

$$= \frac{C}{x}$$

$$= \frac{x^2 + 4x + 4}{x}$$

$$= x + 4 + \frac{4}{x}$$

Marginal cost when x = 7

$$= \frac{dC}{dx}$$

$$= 2x + 4$$

Put x = 7

$$= 14 + 4$$

$$= 18$$

15. the cost of producing x – articles is given by
$$C = x^2 + 15x + 81$$
Find the average cost and marginal cost functions . Find the marginal cost when x = 10 . Find x for which the marginal cost equals average cost

SOLUTION :

$C = x^2 + 15x + 81$

Average Cost

$$= \frac{C}{x}$$

$$= \frac{x^2 + 15x + 81}{x}$$

$$= x + 15 + \frac{81}{x}$$

Marginal cost when x = 10

$$= \frac{dC}{dx}$$

$$= 2x + 15$$

Put x = 10

$$= 20 + 15$$

$$= 35$$

For what value of x ,
Marginal cost   =   average cost

$$2x + 15 = x + 15 + \frac{81}{x}$$

$$x = \frac{81}{x}$$

$$x^2 = 81$$

$$x = 9$$

(output x cannot be negative)

16. if the total cost function is given by ;
$$C = 5x^3 + 2x^2 + 7$$
Find the average cost and the marginal cost when x = 4

SOLUTION :

$C = 5x^3 + 2x^2 + 7$

Average Cost when x = 4

$$= \frac{C}{x}$$

$$= \frac{5x^3 + 2x^2 + 7}{x}$$

$$= 5x^2 + 2x + \frac{7}{x}$$

Put x = 4

$$= 5(16) + 2(4) + \frac{7}{4}$$

$$= 80 + 8 + \frac{7}{4}$$

$$= 88 + \frac{7}{4}$$

$$= \frac{352 + 7}{4}$$

$$= 359/4$$

Marginal cost when x = 4

$= \dfrac{dC}{dx}$

$= 15x^2 + 4x$

   Put x = 4

$= 15(16) + 4(4)$

$= 240 + 16$

$= 256$

17.   the total cost of x pencils is given by

     $C = 15 + 28x - x^2$

Find x when the marginal cost is 20 . Find the average cost at this value of x

SOLUTION :

Marginal cost   = 20

       $\dfrac{dC}{dx}$ = 20

     28 − 2x = 20

          8 = 2x

         x = 4

Average cost at x = 4

$= \dfrac{C}{x}$

$= \dfrac{15 + 28x - x^2}{x}$

$= \dfrac{15 + 28 - x}{x}$

   Put x = 4

$= \dfrac{15 + 28 - 4}{4}$

$= \dfrac{15 + 24}{4}$

$= \dfrac{15 + 96}{4}$

$= \dfrac{111}{4}$

18.   the total cost of producing n note books is given by

     $C = 1500 - 75n + 2n^2 + \dfrac{n^3}{5}$

Find the marginal cost at n = 10

SOLUTION :

Marginal cost at n = 10

$= \dfrac{dC}{dn}$

$= -75 + 4n + \dfrac{3n^2}{5}$

   Put n = 10

$= -75 + 40 + \dfrac{3(100)}{5}$

$= -75 + 40 + 60$ $= 25$

19.   the total cost of 't' toy cars is given by

     $C = 5(2^t) + 17$

Find the marginal cost and average cost at t = 3

SOLUTION :

Marginal Cost at t = 3

$= \dfrac{dC}{dt}$

$= 5(2^t . \log 2)$

   Put t = 3

$= 5(2^3 . \log 2)$

$= 5(8 . \log 2)$

$= 40\log 2$

Average cost at t = 3

$= \dfrac{C}{t}$

$= \dfrac{5(2^t) + 17}{t}$

   Put t = 3

$= \dfrac{5(2^3) + 17}{3}$

$= \dfrac{40 + 17}{3}$

$= \dfrac{57}{3}$

$= 19$